Oxford Cambridge and RSA

Wednesday 13 October 2021 – Afternoon

A Level Mathematics B (MEI)

H640/02 Pure Mathematics and Statistics

Time allowed: 2 hours

8979677975

You must have:
- the Printed Answer Booklet
- a scientific or graphical calculator

INSTRUCTIONS

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION

- The total mark for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **12** pages.

ADVICE

- Read each question carefully before you start your answer.

OCR is an exempt Charity

Formulae A Level Mathematics B (MEI) (H640)

Arithmetic series

$$S_n = \tfrac{1}{2}n(a+l) = \tfrac{1}{2}n\{2a+(n-1)d\}$$

Geometric series

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \text{ for } |r| < 1$$

Binomial series

$$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \ldots + {}^nC_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$$

where $^nC_r = {}_nC_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x| < 1,\ n \in \mathbb{R})$$

Differentiation

$\mathrm{f}(x)$	$\mathrm{f}'(x)$
$\tan kx$	$k\sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

Quotient Rule $y = \dfrac{u}{v}$, $\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$

Differentiation from first principles

$$\mathrm{f}'(x) = \lim_{h \to 0} \frac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$$

Integration

$$\int \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}\,dx = \ln|\mathrm{f}(x)| + c$$

$$\int \mathrm{f}'(x)(\mathrm{f}(x))^n\,dx = \frac{1}{n+1}(\mathrm{f}(x))^{n+1} + c$$

Integration by parts $\displaystyle\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$

Small angle approximations

$\sin\theta \approx \theta$, $\cos\theta \approx 1 - \tfrac{1}{2}\theta^2$, $\tan\theta \approx \theta$ where θ is measured in radians

Trigonometric identities

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad \left(A \pm B \neq (k + \tfrac{1}{2})\pi\right)$

Numerical methods

Trapezium rule: $\int_a^b y\,\mathrm{d}x \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

The Newton-Raphson iteration for solving $\mathrm{f}(x) = 0$: $x_{n+1} = x_n - \dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$

Probability

$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$

$\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B \mid A) = \mathrm{P}(B)\mathrm{P}(A \mid B)$ **or** $\mathrm{P}(A \mid B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$

Sample variance

$s^2 = \dfrac{1}{n-1}S_{xx}$ where $S_{xx} = \Sigma(x_i - \bar{x})^2 = \Sigma x_i^2 - \dfrac{(\Sigma x_i)^2}{n} = \Sigma x_i^2 - n\bar{x}^2$

Standard deviation, $s = \sqrt{\text{variance}}$

The binomial distribution

If $X \sim \mathrm{B}(n, p)$ then $\mathrm{P}(X = r) = {}^n\mathrm{C}_r p^r q^{n-r}$ where $q = 1 - p$

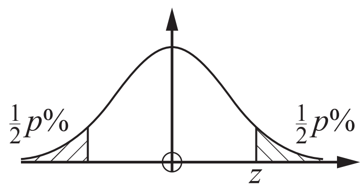
Mean of X is np

Hypothesis testing for the mean of a Normal distribution

If $X \sim \mathrm{N}(\mu, \sigma^2)$ then $\overline{X} \sim \mathrm{N}\left(\mu, \dfrac{\sigma^2}{n}\right)$ and $\dfrac{\overline{X} - \mu}{\sigma/\sqrt{n}} \sim \mathrm{N}(0, 1)$

Percentage points of the Normal distribution

p	10	5	2	1
z	1.645	1.960	2.326	2.576



Kinematics

Motion in a straight line	Motion in two dimensions
$v = u + at$	$\mathbf{v} = \mathbf{u} + \mathbf{a}t$
$s = ut + \frac{1}{2}at^2$	$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$
$s = \frac{1}{2}(u + v)t$	$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$
$v^2 = u^2 + 2as$	
$s = vt - \frac{1}{2}at^2$	$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$

Answer **all** the questions.

Section A (23 marks)

1 The equation of a curve is $y = 4x^2 + 8x + 1$.

The curve is stretched parallel to the x-axis with scale factor 2.

Find the equation of the new curve, giving your answer in the form $y = ax^2 + bx + c$, where a, b and c are integers to be determined. **[2]**

2 **(a)** Write 65° in radians, giving your answer in the form $k\pi$, where k is a fraction in its lowest terms. **[2]**

(b) Write 0.211 radians in degrees, giving your answer correct to **1** decimal place. **[1]**

3 Draw a number line to show the values of x which belong to the set $\{x : x \geqslant 2\} \cap \{x : x < 7\}$. **[3]**

4 Sketch the graph of $y = |2x - 3|$. **[3]**

5 It is known that 40% of people in Britain carry a certain gene.

A random sample of 32 people is collected.

(a) Calculate the probability that exactly 12 people carry the gene. **[1]**

(b) Calculate the probability that at least 8 people carry the gene, giving your answer correct to **3** decimal places. **[2]**

6 You are given that $\mathbf{v} = 2\mathbf{a} + 3\mathbf{b}$, where $\mathbf{a}$ and $\mathbf{b}$ are the position vectors

$\mathbf{a} = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -1 \\ 6 \end{pmatrix}$.

(a) Determine the magnitude of $\mathbf{v}$. **[3]**

(b) Determine the angle between $\mathbf{v}$ and the vector $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$. **[2]**

7 The parametric equations of a circle are

$x = 7 + 5\cos\theta$, $\quad y = 5\sin\theta - 3$, for $0 \leqslant \theta \leqslant 2\pi$.

(a) Find a cartesian equation of the circle. **[3]**

(b) State the coordinates of the centre of the circle. **[1]**

Answer **all** the questions.

Section B (77 marks)

8 The Normal variable X is transformed to the Normal variable Y.

The transformation is $y = a + bx$, where a and b are positive constants.

You are given that $X \sim N(42, 6.8)$ and $Y \sim N(57.2, 11.492)$.

Determine the values of a and b. **[4]**

9 Labrador puppies may be black, yellow or chocolate in colour. Some information about a litter of 9 puppies is given in the table.

	male	female
black	1	3
yellow	2	1
chocolate	1	1

Four puppies are chosen at random to train as guide dogs.

(a) Determine the probability that exactly 3 females are chosen. **[3]**

(b) Determine the probability that **at least** 3 black puppies are chosen. **[3]**

(c) Determine the probability that exactly 3 females are chosen **given that** at least 3 black puppies are chosen. **[3]**

(d) Explain whether the 2 events

‘choosing exactly 3 females’ and ‘choosing at least 3 black puppies’

are independent events. **[1]**

10 Ben has an interest in birdwatching.

For many years he has identified, at the start of the year, 32 days on which he will spend an hour counting the number of birds he sees in his garden.

He divides the year into four using the Meteorological Office definition of seasons. Each year he uses stratified sampling to identify the 32 days on which he will count the birds in his garden, drawn equally from the four seasons.

Ben's data for 2019 are shown in the stem and leaf diagram in **Fig. 10.1**.

0	3 5 9 9 9
1	0 0 1 1 2 4 5 6 7 8 9
2	0 1 4 6 7 8 9
3	0 0 2 3
4	0 3 6
5	1
6	0

Fig. 10.1

(a) Suggest a reason why Ben chose to use stratified sampling instead of simple random sampling. **[1]**

(b) Describe the shape of the distribution. **[1]**

(c) Explain why the mode is not a useful measure of central tendency in this case. **[1]**

(d) For Ben's sample, determine

- the median,
- the interquartile range. **[3]**

Ben found a boxplot for the sample of size 32 he collected using stratified sampling in 2015.

The boxplot is shown in **Fig. 10.2**.

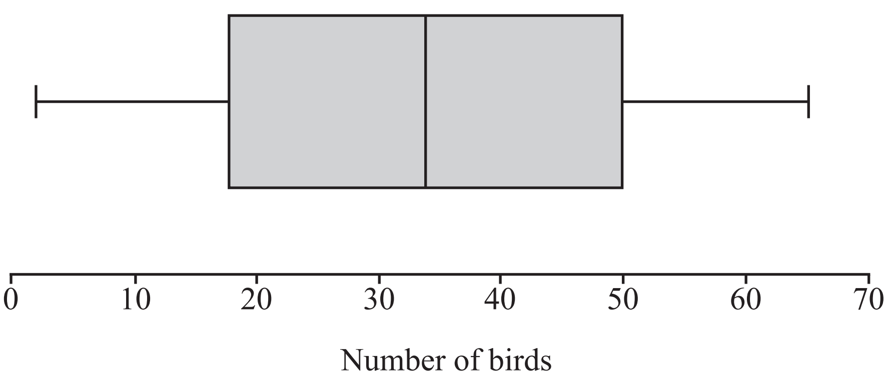


Fig. 10.2

In 2016 Ben replaced his hedge with a garden fence.
Ben now believes that

- he sees fewer birds in his garden,
- the number of birds he sees in his garden is more variable.

(e) With reference to **Fig. 10.2** and your answer to part **(d)**, comment on whether there is any evidence to support Ben's belief. **[2]**

Jane says she can tell that the data for 2015 is definitely uniformly distributed by looking at the boxplot.

(f) Explain why Jane is wrong. **[1]**

11 In 2010 the heights of adult women in the UK were found to have mean $\mu = 161.6$ cm and variance $\sigma^2 = 1.96\,\text{cm}^2$.

It is believed that the mean height of adult women in 2020 in the UK is greater than in 2010.

In 2020 a researcher collected a random sample of the heights of 200 adult women in the UK.

The researcher calculated the sample mean height and carried out a hypothesis test at the 5% level to investigate whether there was any evidence to suggest that the mean height of adult women in the UK had increased.

The researcher assumed that the variance was unaltered.

(a)
- State suitable hypotheses for the test, defining any variables you use.
- Explain whether the researcher conducted a 1-tail or a 2-tail test. **[3]**

(b) Determine the critical region for the test. **[2]**

The researcher found that the sample mean was 161.9 cm and made the following statements.

- The sample mean is in the critical region.
- The null hypothesis is accepted.
- This proves that the mean height of adult women in the UK is unaltered at 161.6 cm.

(c) Explain whether each of these statements is correct. **[3]**

12 **Fig. 12.1** shows an excerpt from the pre-release material.

	A	B	C	D	E	F	G	H
1	**Sex**	**Age**	**Marital**	**Weight**	**Height**	**BMI**	**Waist**	**Pulse**
2	Female	34	Married	60.3	173.4	20.05	82.5	74
3	Female	85	Widowed	64.7	161.2	24.9	#N/A	#N/A
4	Female	48	Divorced	100.6	171.4	34.24	105.6	92
5	Male	61	Married	70.9	169.5	24.68	92.2	70
6	Male	68	Divorced	96.8	181.6	29.35	112.9	68

Fig. 12.1

There was no data available for cell H3.

(a) Explain why #N/A is used when no data is available. **[1]**

Fig. 12.2 shows a scatter diagram of pulse rate against BMI (Body Mass Index) for females. All the available data was used.

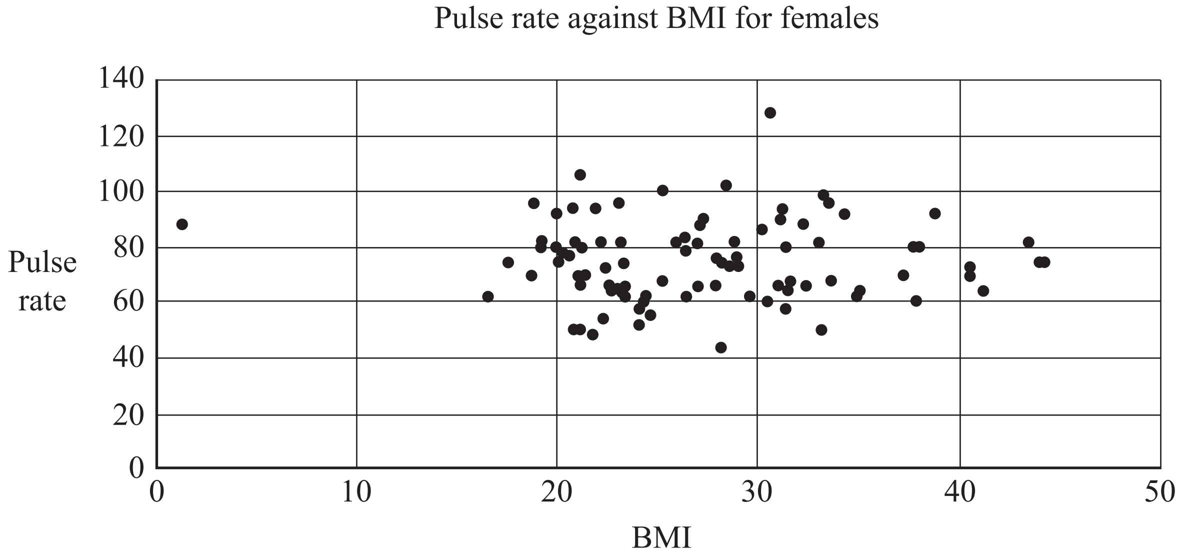


Fig. 12.2

There are **two** outliers on the diagram.

(b) On the copy of **Fig. 12.2** in the Printed Answer Booklet, ring these outliers. **[1]**

(c) Use your knowledge of the pre-release material to explain whether either of these outliers should be removed. **[2]**

(d) State whether the diagram suggests there is any correlation between pulse rate and BMI. **[1]**

The product moment correlation coefficient between **waist measurement**, w, in cm and **BMI**, b, for females was found to be 0.912. All the available data was used.

(e) Explain why a model of the form $w = mb + c$ for the relationship between waist measurement and BMI is likely to be appropriate. **[1]**

The LINEST function on a spreadsheet gives $m = 2.16$ and $c = 33.0$.

(f) Calculate an estimate of the value for cell G3 in **Fig. 12.1**. **[1]**

13 At a certain factory Christmas tree decorations are packed in boxes of 10.

The quality control manager collects a random sample of 100 boxes of decorations and records the number of decorations in each box which are damaged.

His results are displayed in **Fig. 13.1**.

Number of damaged decorations	0	1	2	3	4	5 or more
Number of boxes	19	35	28	13	5	0

Fig. 13.1

(a) Calculate

- the mean number of damaged decorations per box,
- the standard deviation of the number of damaged decorations per box. **[2]**

It is believed that the number of damaged decorations in a box of 10, X, may be modelled by a binomial distribution such that $X \sim \mathrm{B}(n, p)$.

(b) State suitable values for n and p. **[1]**

(c) Use the binomial model to complete the copy of **Fig. 13.2** in the Printed Answer Booklet, giving your answers correct to **1** decimal place. **[3]**

Number of damaged decorations	0	1	2	3	4	5 or more
Observed number of boxes	19	35	28	13	5	0
Expected number of boxes						

Fig. 13.2

(d) Explain whether the model is a good fit for these data. **[1]**

14 The equation of a curve is

$$y = x^2(x-2)^3.$$

(a) Find $\frac{dy}{dx}$, giving your answer in factorised form. **[4]**

(b) Determine the coordinates of the stationary points on the curve. **[4]**

In part **(c)** you may use the result $\frac{d^2y}{dx^2} = 4(x-2)(5x^2-8x+2)$.

(c) Determine the nature of the stationary points on the curve. **[3]**

(d) Sketch the curve. **[2]**

15 **(a)** Show that $\sum_{r=1}^{\infty} 0.99^{r-1} \times 0.01 = 1$. **[3]**

Kofi is a very good table tennis player. Layla is determined to beat him.

Every week they play one match of table tennis against each other. They will stop playing when Layla wins the match for the first time.

X is the discrete random variable "the number of matches they play in total".

Kofi models the situation using the probability function

$$\mathrm{P}(X = r) = 0.99^{r-1} \times 0.01 \quad r = 1, 2, 3, 4, \ldots$$

Kofi states that he is 95% certain that Layla will not beat him within 6 years.

(b) Determine whether Kofi's statement is consistent with his model. **[3]**

In between matches, Layla practises, but Kofi does not.

(c) Explain why Layla might disagree with Kofi's model. **[1]**

Layla models the situation using the probability function

$$\mathrm{P}(X = r) = kr^2 \qquad r = 1, 2, 3, 4, 5, 6, 7, 8.$$

(d) Explain how Layla's model takes into account the fact that she practises between matches, but Kofi's does not. **[1]**

Layla states that she is 95% certain that she will beat Kofi within the first 6 matches.

(e) Determine whether Layla's statement is consistent with her model. **[3]**

16 In this question you must show detailed reasoning.

Find $\int \frac{x}{1+\sqrt{x}}\,\mathrm{d}x$. **[8]**

END OF QUESTION PAPER